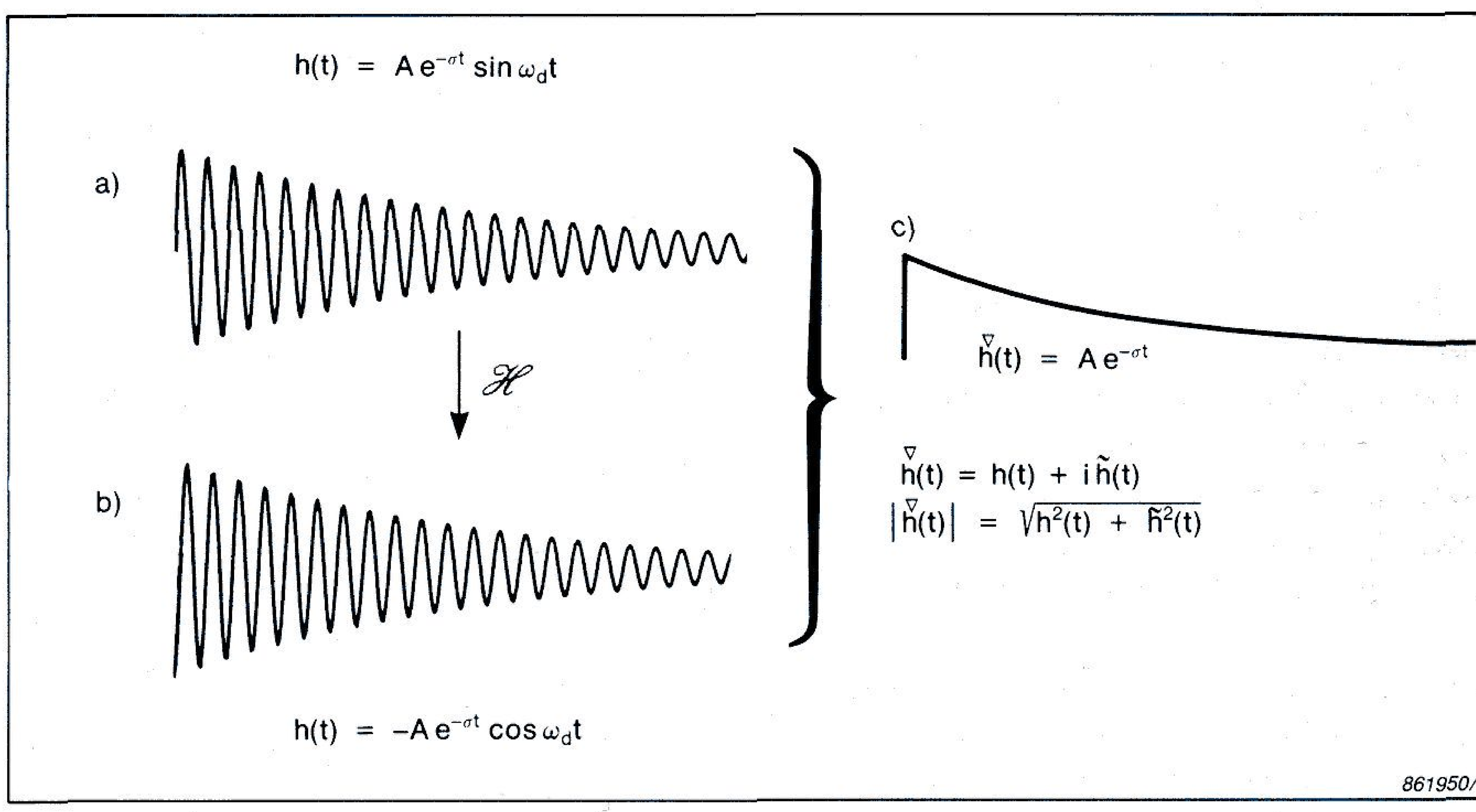
Brüel & Kjær application notes

Practical use of the "Hilbert transform"

The Brüel & Kjær Dual-Channel Signal Analyzers Type 2032 and 2034 implement the Hilbert transform to open up new analysis possibilities in the time domain. By means of the Hilbert transform, the envelope of a time signal can be calculated, and displayed using a logarithmic amplitude scale enabling a large display range. Two examples which use the Hilbert transform are presented here:

- The determination of the damping or decay rate at resonances, from the impulse-response function.
- The estimation of propagation time, from the cross-correlation function.



The Hilbert transform enables computation of the envelope of the impulse-response function

The envelope

Many application measurements result in a time signal containing a rapidly-oscillating component. The amplitude of the oscillation varies slowly with time, and the shape of the slow time-variation is called the "envelope". The envelope often contains important information about the signal. By using the Hilbert transform, the rapid oscillations can be removed from the signal to produce a direct representation of the envelope alone.

For example, the impulse response of a single degree-of-freedom system is an exponetially-damped sinusoid, h (t). This is shown as (a) in the above figure. The envelope of the signal is determined by the decay rate σ.

The Hilbert transform, $\mathscr{H}$ is used to calculate a new time signal $\tilde{h}(t)$ from the original time signal h (t). The time signal $\tilde{h}(t)$ is a cosine function whereas h (t) is a sine, both are shown above.

The magnitude of the *analytic signal* $\overset{\nabla}{h}(t)$ can be directly calculated from h and $\tilde{h}$. The magnitude of $\overset{\nabla}{h}(t)$ is the envelope of the original time signal and is shown above as (c). It has the following advantages over h (t):

1. Removal of the oscillations allows detailed study of the envelope.
2. Since $\overset{\nabla}{h}(t)$ is a positive function, it can be graphically represented using a logarithmic amplitude scale to enable a display range of 1:10 000, or more. The original signal, h(t), includes both positive and negative values and is traditionally displayed using a linear amplitude scale. This limits the display range to about 1:100.

Decay-rate estimation

Determining the frequency and corresponding damping at resonances is often the first step in solving a vibration problem for a structure. Figure 1 shows the log. magnitude of a mechanical-mobility measurement. Within the excitation frequency range of 0 Hz to 3,2 kHz, five resonances are clearly seen. The resonance frequencies can be read directly with an accuracy determined by the resolution of the analysis, i.e. 4 Hz. The decay rate at the resonances is often determined by the half-power (or 3 dB) bandwidth, B, of the resonance peak. $B_{3dB} = 2\sigma$. In this

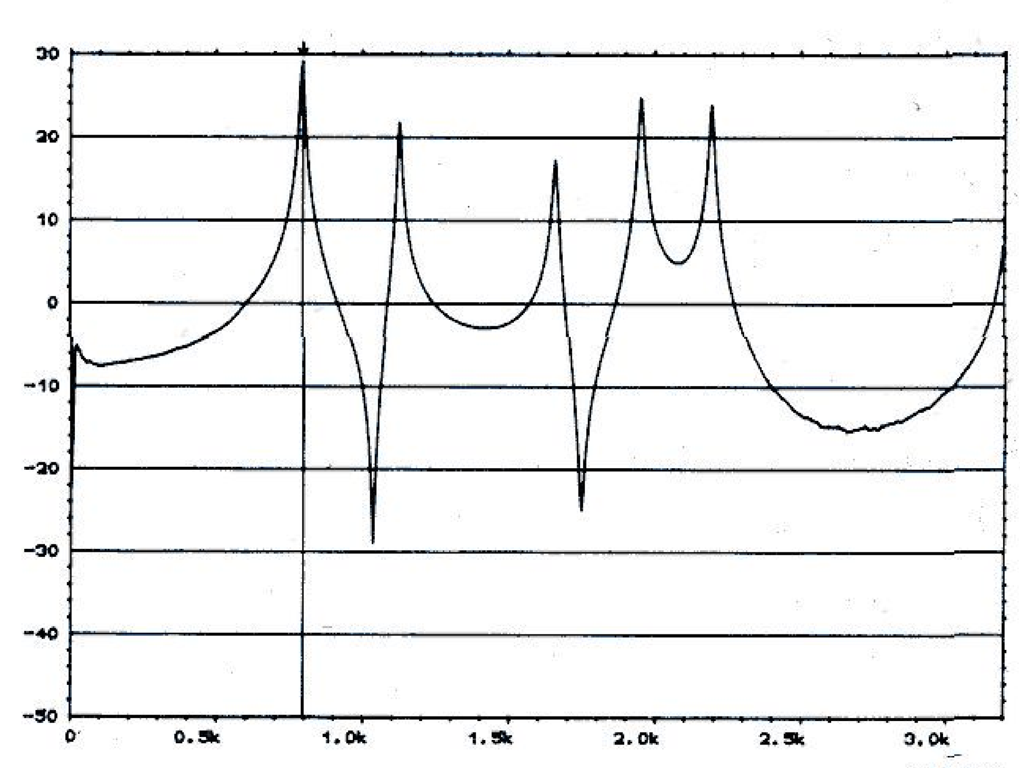


Fig. 1.

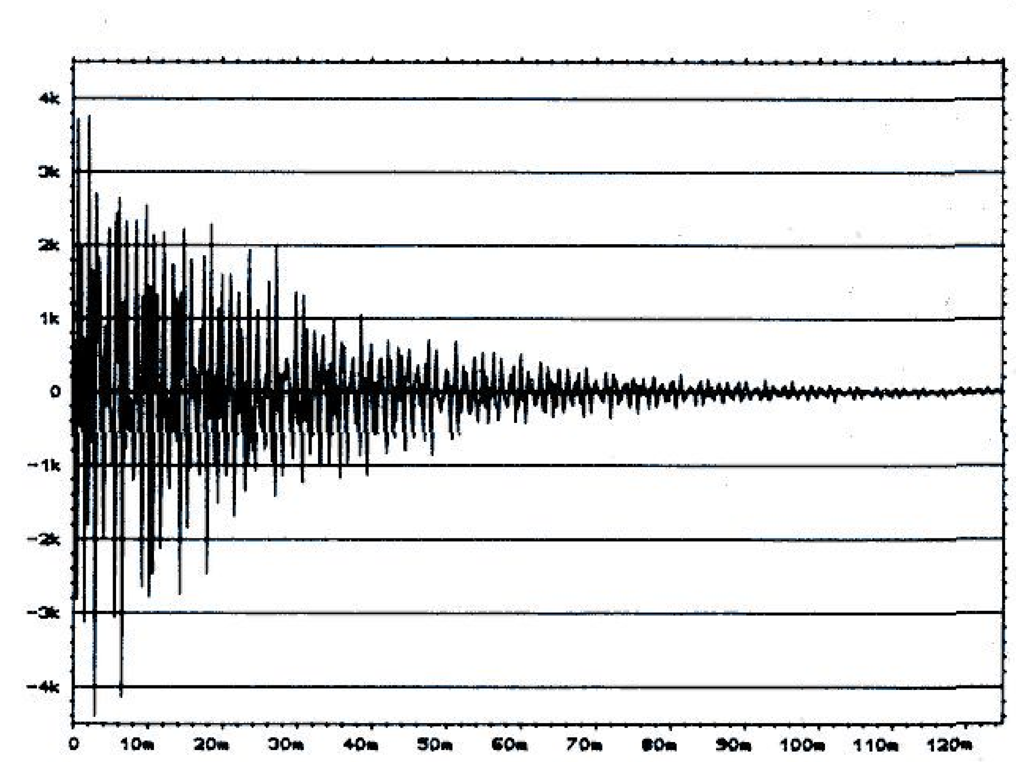


Fig. 2.

case B is of the order of the resolution, consequently a determination of the B_{3dB} (and hence σ) will be very inaccurate. Two methods can be used to obtain a more accurate estimate of the damping:

1. A zoom-analysis using a much smaller Δf. This involves a new analysis for each resonance, making five new measurements in total.
2. The damping at each resonance can be determined from the envelope of the associated impulse-response function. This method is illustrated in Figs. 1 to 6, from which σ for each resonance can be easily found from the original measurement.

Figure 1 shows the frequency-response function, and Fig. 2 shows the corresponding impulse-response function. However, this cannot be used to calculate σ, as it contains five exponentially-damped sinusoids (one for each resonance) superimposed.

Figure 3 shows a single resonance which has been isolated using the frequency weighting facility of the 2032/2034. The corresponding impulse-response function, shown in Fig. 4, clearly shows the exponential decaying sinusoid.

Figure 5 shows the magnitude of the analytic signal of the impulse-response function on a linear amplitude scale. By using a log. amplitude axis, the envelope is a straight line, see Fig. 6. The analyzer's reference-cursor is used to measure the time constant τ corresponding to an amplitude decay of 8,7 dB. From τ, the decay rate and hence the damping of the resonance can be calculated directly ($\sigma = 1/\tau$).

By using the Hilbert transform, it is possible to determine the decay rate for the five individual resonances, without having to make new, more narrowly-banded measurements.

Propagation-Time estimation

The propagation time (from point A to B) of a signal is usually estimated by measuring the signal at A and B, and calculating the cross-correlation

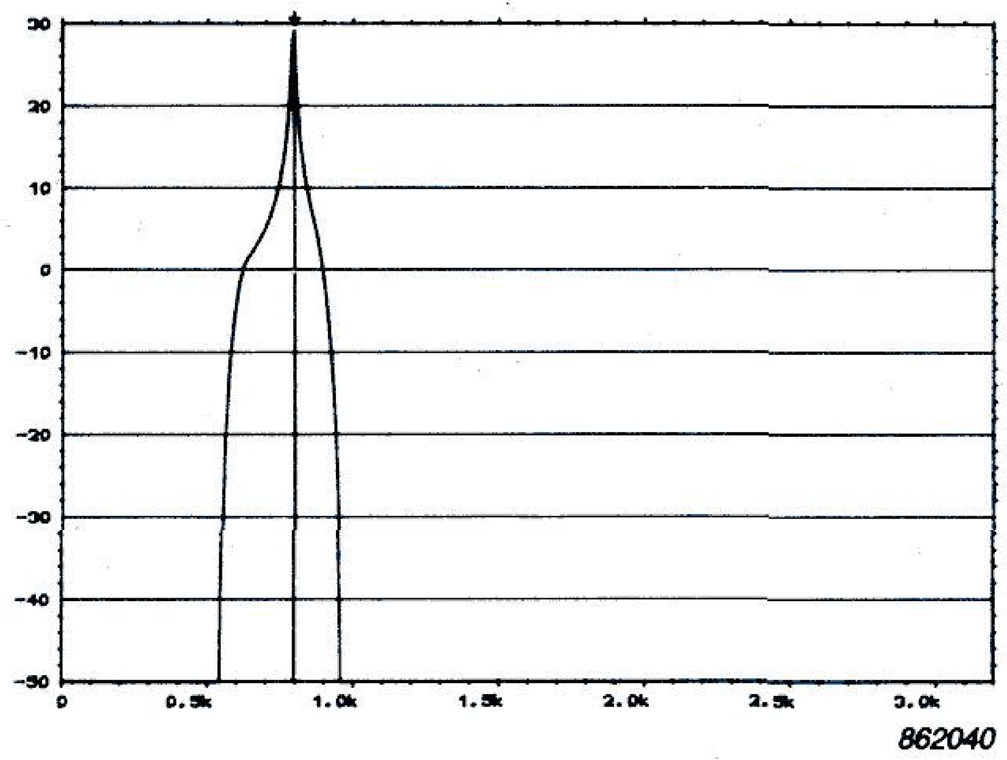


Fig. 3.

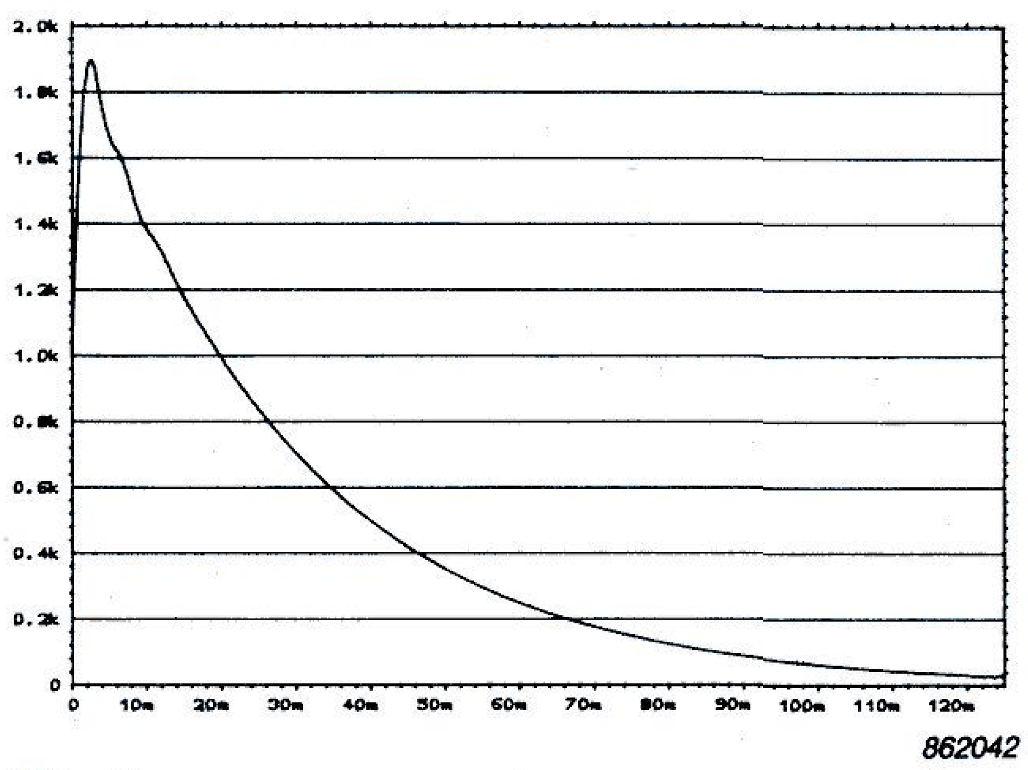


Fig. 5.

function R_{AB} (t). Figure 7 shows an example for such a measurement, R_{AB} $(t)_{MAX}$ indicates the propagation time. The maximum is found at 3 ms delay which corresponds to a 1 m sound propagation in air.

However, if a zoom measurement is made, the result shown in Fig. 8 could be observed. Now R_{AB} $(t)_{MAX}$ doesn't correspond to 3 ms. In addition, the question arises: Which peak should be used to determine the propagation time, the positive or negative? The answer is neither. Instead the maximum of the envelope should be used, as this will always indicate the correct propagation time, see Fig. 9.

By using the Hilbert transform, the correct propagation time can easily be found from the envelope of the cross-correlation function, whether or not the peak of R_{AB} (t) corresponds to the envelope maximum.

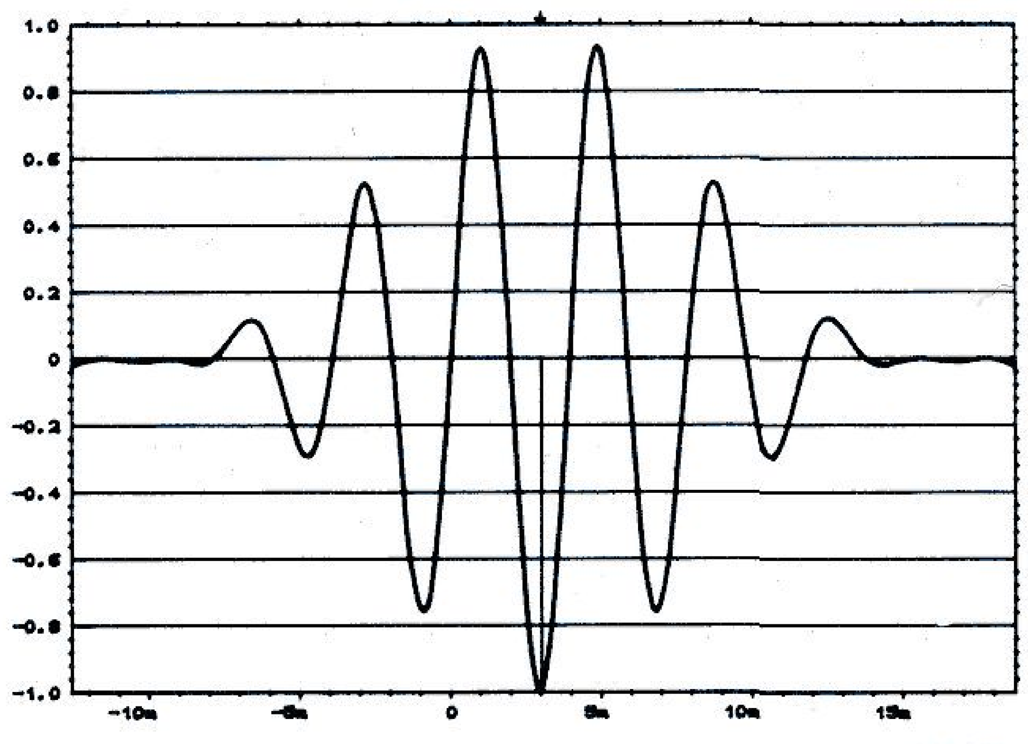


Fig. 7.

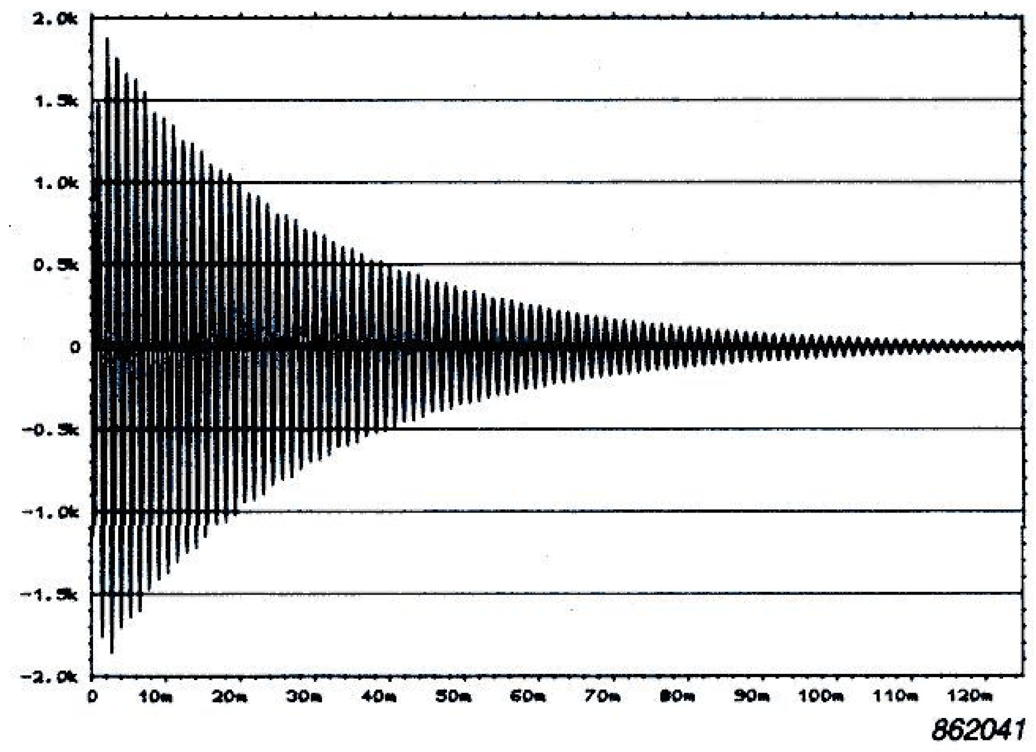


Fig. 4.

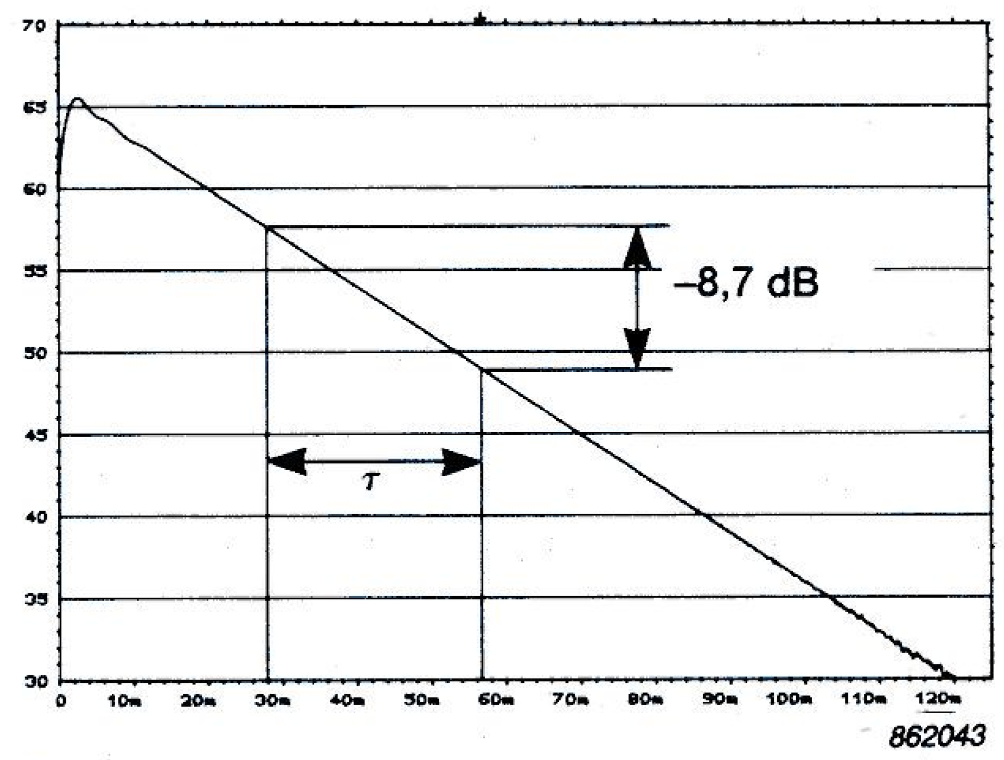


Fig. 6.

References

A short discussion of the Hilbert transform can be found in ref. 1, while ref. 2 discusses the properties and applications of the Hilbert transform.

1. N. Thrane "The Hilbert transform", Technical Review No. 3 1984, Brüel & Kjær.
2. J.S. Bendat, "The Hilbert transform and Applications to Correction Measurements", Brüel & Kjær, 1985.

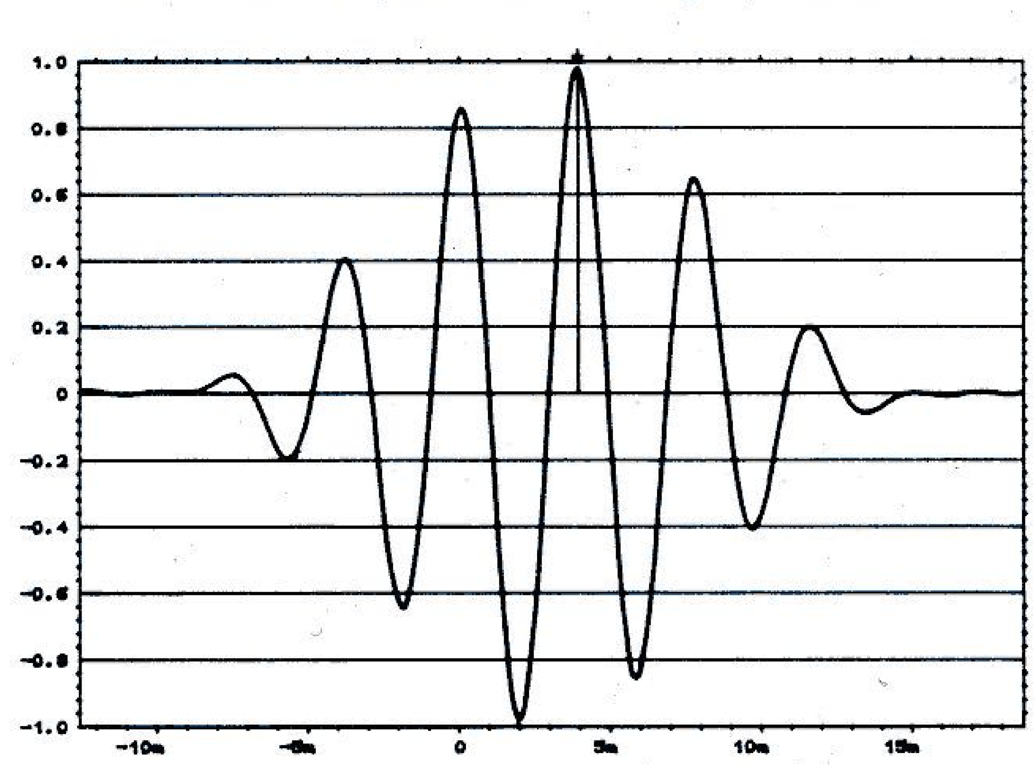


Fig. 8.

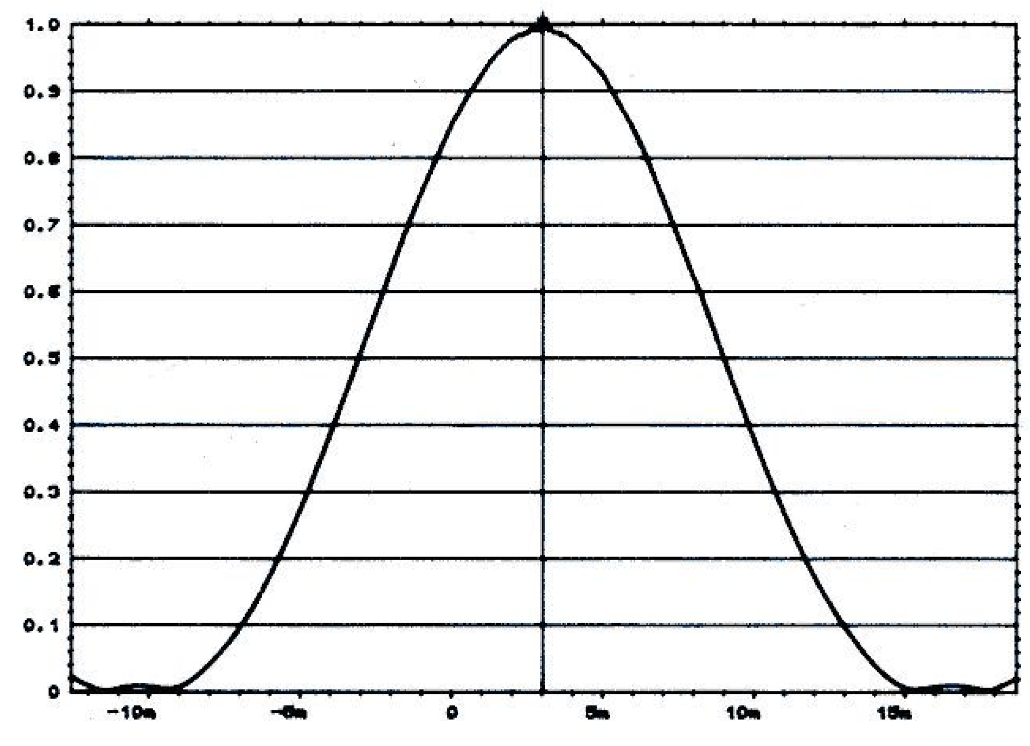


Fig. 9.

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